MATHEMATICS

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MATHEMATICS
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VECTOR ALGEBRA

& Their Properties

CONTENTS

THINGS TO REMEMBER

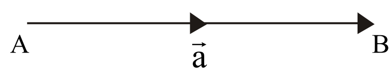
✷ Scalar and Vector Quantities

Those quantities which have only magnitude and are not related to any direction in space, also does not follow the rules of vactor algebra, are called scalar quantities. eg, mass, speed, distance etc.

Those quantities which have both magnitude and direction and follow the rules of vector algebra are called vector quantities. eg, displacement, velocity etc.

Representation of a Vector

A vector is generally represented by a direction line segment, say $\overrightarrow{AB}$. A is called the initial oint and B s called the terminal point. The positive number representing the measure of the length of the line segment denoting the vector is called the modulus of magnitude of the vector and the arrow head indicates its direction. The modulus of vector $\overrightarrow{AB}$ is denoted $|\overrightarrow{AB}|$.



✷ Types of Vectors

1. Zero Vector

A vector of zero magnitude ie, which has the same initial and terminal point, is called a zero vector. It is denoted by $\vec{O}$.

2. Unit Vector

A vector of zero magnitude in the direction of a vector $\vec{a}$ is called a unit vector along $\vec{a}$ and is denoted by $\hat{a}$.

Symbolically, $$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

3. Equal Vector

If magnitude and direction of two vectors are same, then those are called equal vectors.

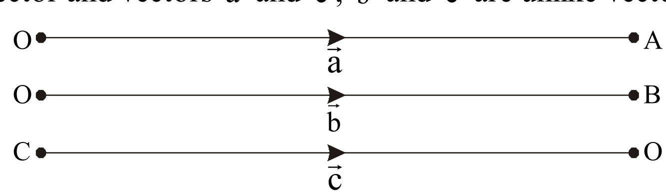
4. Parallel Vector

Two vectors $\vec{a}$ and $\vec{b}$ are said to be parallel if either both have same line of action or support of there exists a scalar λ such that if $\lambda < 0$, then $\vec{a}$ and $\vec{b}$ have same direction or sense. If $\lambda < 0$, then $\vec{a}$ and $\vec{b}$ have opposite direction or sense.

5. Like and Unlike Vector

Two parallel vectors are said to be like when they have same sense of direction ie, angle between them is zero. Otherwise vectors are said to be unlike vectors and angle between them is π.

Vector $\vec{a}$ and $\vec{b}$ are like vector and vectors $\vec{a}$ and $\vec{c}$, $\vec{b}$ and $\vec{c}$ are unlike vectors.



6. Negative Vector

A vector having the same magnitude as that of a given vector and direction opposite to that given vector is called negative vector.

7. Coinitial and Coterminal Vector

The vector which have the same initial point are co-initial vectors. Similarly, the vectors which have the same terminal point are called co-terminal vector.

8. Free Vectors

The vector whose initial point or tail is not fixed is called free or non-localised vector.

9. Collinear Vectors

Two or more vectors are known as collinear vectors, if they are parallel to a given straight line. The magnitude of collinear vectors can be diferent.

10. Coplanar Vectors

Vectors are said to be coplanar, if they occur in same of common plane.

11. Position Vectors

Let O be a fixed origin, then the position vector of a point P is the vector . If $\vec{a}$ and $\vec{b}$ are position vectors of two points A and B, then $\overrightarrow{AB} = \vec{b} - \vec{a}$.

= position vector of $\vec{b}$ – position vectors of $\vec{a}$.

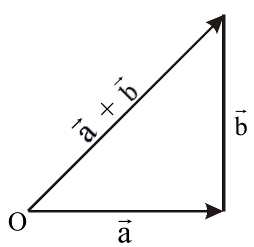
✷ Addition of Vectors

The addition of two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} + \vec{b}$, and it is known as resultant of $\vec{a}$ and $\vec{b}$.

There are three methods of addition of vectors.

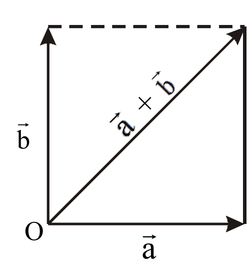
1. Triangle Law

If two vectors $\vec{a}$ and $\vec{b}$ lie along the two sides of a triangle in consecutive order (as shown in the adjoining figure), then third side represents the sum (resultant) $\vec{a} + \vec{b}$.



2. Parallelogram Law

If two vectors lie along two adjacent sides of a parallelogram (as shown in the adjoining figure) then diagonal of the parallelogram through the common vertex represents their sum.



3. Polygon Law

If $(n-1)$ sides of a polygon represent vector $\vec{a}_1, \vec{a}_2, \vec{a}_3$,.........., in consecutive order, then nth side represents their sum (as shown in the adjoining figure).

Properties of vector Addition

(i) Vector addition is commutative, ie,

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

(ii) Vector addition is associative, ie,

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

(iii) $\vec{a} + \vec{O} = \vec{a} = \vec{O} + \vec{a}$, where $\vec{O}$ is additive identify.

(iv) $\vec{a} + (-\vec{a}) = \vec{O} = (-\vec{a}) + \vec{a}$, where $(-\vec{a})$ is additive inverse.

✷ <u>**Multiplication of a Vector by a Scalar**</u>

If a is $\vec{a}$ vector and λ is a scalar, then $\lambda\,\vec{a}$ is a vector parallel to $\vec{a}$ whose modulus is $|\lambda|$ times that of $\vec{a}$. This multiplication is called scalar multiplication.

Properties of Multiplication of a Vector by a Scalar

If λ and μ be two scalars, then

(i) $\lambda(\mu\vec{a}) = \lambda\mu\vec{a}$

(ii) $(\lambda + \mu)\vec{a} = \lambda\vec{a} + \mu\vec{a}$

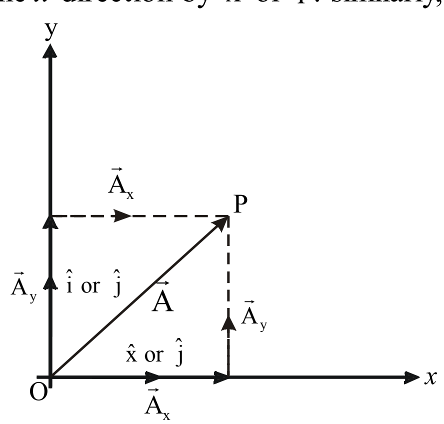
(iii) $\lambda(\vec{a} + \vec{b}) = \lambda\vec{a} + \lambda\vec{b}$

✷ <u>**Component of a Vector**</u>

The process of splitting a vector is called resolution of a vector. In simpler language it would mean, determining the effect of a vector in a particular direction. The parts of the vector obtained after splitting the vectors are known as the components of the venctor.

Component of a vector in Two Dimension

We can define a unit vector in the x-direction by $\hat{x}$ or $\hat{i}$. similarly, in the y- direction we use $\hat{y}$ or $\hat{j}$.



$\vec{A}_x$ is the resolved part of $\vec{A}$ along x-axis and is the projection of $\vec{A}$ on x-axis. Similarly, $\vec{A}_y$ is the resolved part of $\vec{A}$ along y-axis and is the projection of $\vec{A}$ on y-axis.

By the triangle law,

$$\overrightarrow{OP} = \overrightarrow{ON} + \overrightarrow{NP}$$

or $$\vec{A} = \vec{A}_x + \vec{A}_y$$

If A_x and A_y are the magnitudes of $\vec{A}_x$ and $\vec{A}_y$ then $\vec{A}_x\,\hat{i}$ and $\vec{A}_y\,\hat{j}$ are the vector components of $\vec{A}$ in the *x* and y directions respectively.

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

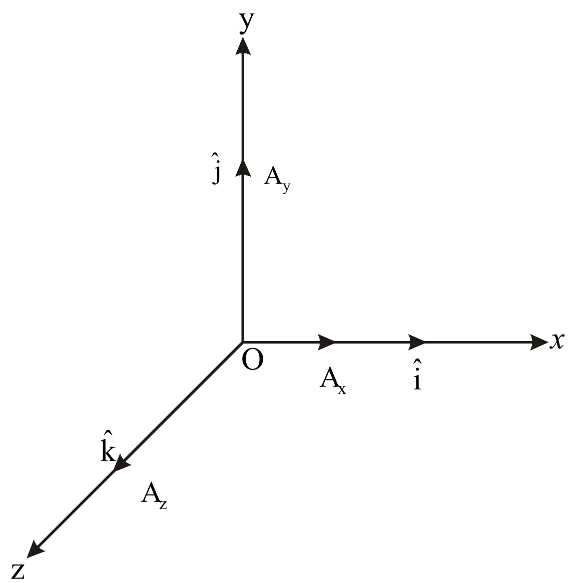
and $$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

Components of a Vector is Three Dimension

Same as in two dimension, in three dimension we can define a unit vector in the *x*-direction by $\hat{i}$ or $\hat{x}$, in y- direction by $\hat{j}$ or $\hat{y}$ and z-direction by $\hat{k}$ or $\hat{z}$. The vector A can be represented by

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

$$= A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$



Where A_x, A_y, and A_z are the magnitudes of $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$ resectively.

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

The unit vector $\hat{i}$, $\hat{j}$, $\hat{k}$ are known as an orthonormal traid of vectors.

✷ **<u>Linear Combination of a Vector</u>**

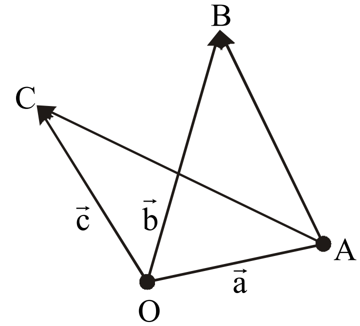
If some vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots\ldots, v_n$ vn can be written as a vector $a_1\vec{v}_1 + a_2\vec{v}_2 + a_3\vec{v}_3 + \ldots\ldots + a_n\vec{v}_n$, where a_1, a_2, a_3,..........,a_n are scalars then this vector is known as linear combination of vectors $\vec{v}_1, \vec{v}_2, \ldots\ldots\ldots, \vec{v}_3$.

If $a_1\vec{v}_1 + a_2\vec{v}_2 + a_3\vec{v}_3 + \ldots\ldots + a_n\vec{v}_n = 0$, thn combination of vectors is known as linearly dependent combination of vectors.

✷ Collinearity of Point or Vectors

Let the position vectors of three points are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{a}+\lambda\vec{b}+\mu\vec{c}=0$, where $1 + \lambda + \mu = 0$, then three points will be collinear.

Three points representing three position vectors can be represent two vectors in the plane.



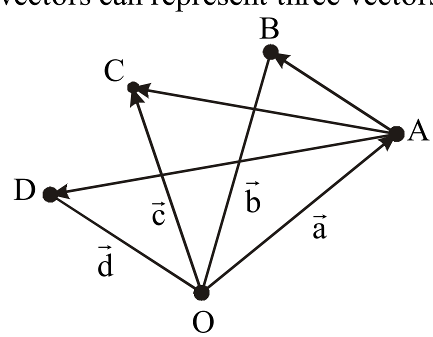
Let A, B and C are three points represented by three position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Now, these three position vectors can represent two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ as shown in figure.

Now, the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear, if there exists a linear relation between the two, such that $\overrightarrow{AB} = \lambda\overrightarrow{AC}$.

Collinearity of Four Points

Let A, B, C and D be four points represented by four position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.

Now, these four position vectors can represent three vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$.



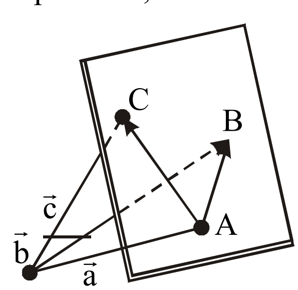
The vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ are collinear, if.

$\overrightarrow{AC} = m\overrightarrow{AB}$ and $\overrightarrow{AB} = n\overrightarrow{AB}$ and similar other cases for more than four vectors.

✷ Coplanarity of Points or Vectors

Coplanarity of Three Points

Three points A, B and C represented by position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and from the figure, two vectors, are alwas coplanar ie, two vectors always form their own plane.



Coplanarity of Four Points

The necessary and sufficient condition that four points with position vectors, $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ should be coplanar is that there exist four scalars x,y,z,t not all zero, such that

$$x\vec{a} + y\vec{b} + z\vec{c} + t\vec{d} = 0 \qquad x + y + z + t = 0$$

To prove that the four points A, B, C and D having position vectors as $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are coplanar.

Step I Find the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ having the reference points as A.

Step II Express one of these vectors as the linear combination of the other two.

$$\overrightarrow{AB} = \lambda \overrightarrow{AC} + \mu \overrightarrow{AD}$$

Step III Now, compare the coefficients on LHS and RHS in respective manner and thus find the respective values of λ and μ. d

Step IV If real values of the scalars λ and μ exist, then the three vectors representing four points are coplanar otherwise not.

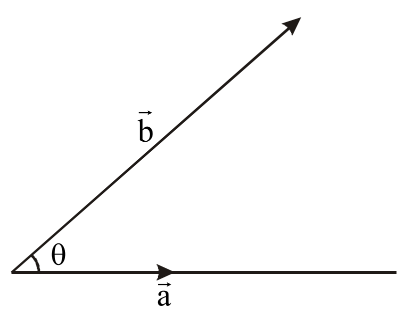
✷ **Product of Two Vectors**

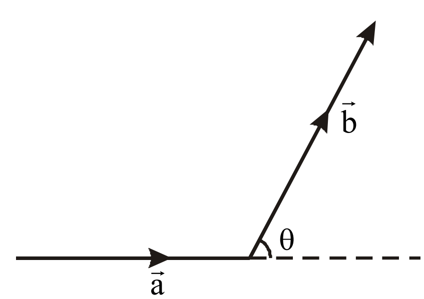
There are two methods of products of two vectors.

1. **Scalar Product or Dot Product**

Let a and b be two non-zero vectors inclined at an angle θ. Then the scalar product of $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}.\vec{b}$ and defined as

$$\vec{a}.\vec{b} = ab\cos\theta,\ 0 \le \theta \le \pi$$





Since, scalar production of two vectors is a scalar quantity so it is called scalar product.

Properties of Scalar Product

(i) The scala producto of two vectors is commutative is, $\vec{a}.\vec{b} = \vec{b}.\vec{a}$.

(ii) If m and n be any two sclars and a and b be any two vectors, then $(m\vec{a}).(n\vec{b}) = (n\vec{a}).(m\vec{b})$

(iii) $\vec{a}(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$ (Distributive law)

(iv) $\vec{a}.\vec{a} = |\vec{a}|^2$

(v) If two vectors a and are perpendicular to each other, then $\vec{a}.\vec{b} = 0$.

(a) $\vec{a}.\vec{b} < 0$, iff $\vec{a}$ and $\vec{b}$ are inclined at an obtuse angle.

(b) $\vec{a}.\vec{b} > 0$, iff $\vec{a}$ and $\vec{b}$ are inclined at an acute angle.

(vi) If $\vec{a}\cdot\vec{b} = 0$ then either $\vec{a} = 0$, $\vec{b} = 0$ or $\vec{a} \perp \vec{b}$

(vii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

(viii) $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

(ix) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

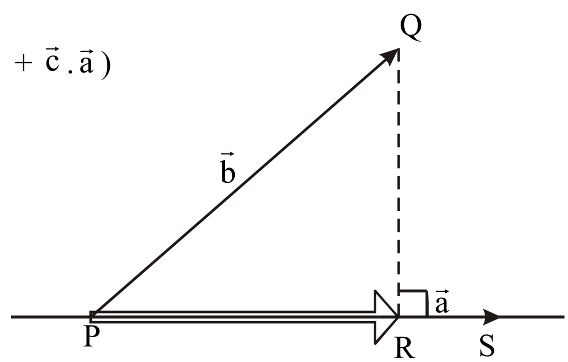
$$\cos\theta = \left[\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right] = \left[\frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}\right]$$

If a1a2 + b1b2 + c1c2 = 0, then both vectors are perpendicular to each other and if $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$, then both vectors are parallel to each other.

(x) $(\vec{a} + \vec{b} + \vec{c}) = a^2 + b^2 + c^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

(xi) Projection of $\vec{b}$ along $\vec{a} = \frac{\vec{b}.\vec{a}}{|\vec{a}|}$.

and projection of $\vec{a}$ along $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$.



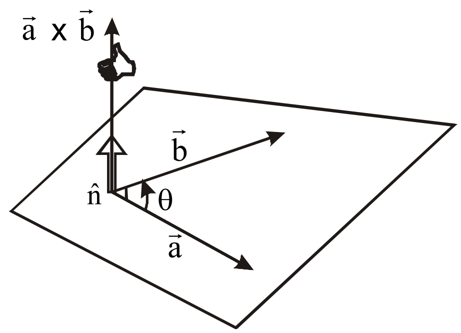
If b represents a force, then projection of $\vec{b}$ along $\vec{a}$ represents the effective force in the direction of $\vec{a}$.

Total work done = $\vec{F}\cdot\vec{d}$ = F d cos θ where $\vec{F}$ is the forec and $\vec{d}$ is the displacement.

2. Vector Product or Cross Product

The vector product of two vectors $\vec{a}$ and $\vec{b}$ is a vector and is given by

$$\vec{a} \times \vec{b} = ab\sin\theta\,\vec{n}$$



Where t be the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is a perpendicular unit vector to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\hat{n}$ form a right handed system.

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

Also, $\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$, Where, n indicatedes direction of $\vec{a} \times \vec{b}$.

Properties of Vector Product

(i) Vector product is not commtative, ie,

$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$$

But $\qquad \vec{b} \times \vec{a} \neq -\vec{a} \times \vec{b}$

(ii) If *m* and *n* be two scalars and $\vec{a}$ and $\vec{b}$ be two vectors, then

$$(m\vec{a}) \times (n\vec{b}) = mn(\vec{a} \times \vec{b}) = (n\vec{a}) \times (m\vec{b})$$

(iii) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ (distributive law)

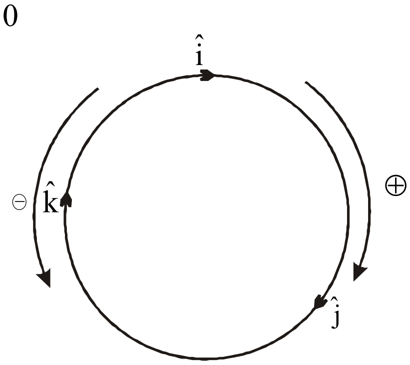
(iv) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a} \times \vec{b} = [(a_2b_3 - a_3b_2)\hat{i} + (a_3b_1 - a_1b_3)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}]$

or $\qquad \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(v) $\vec{a} \times \vec{b} = 0$

If $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or a and b are two collinear vectors.

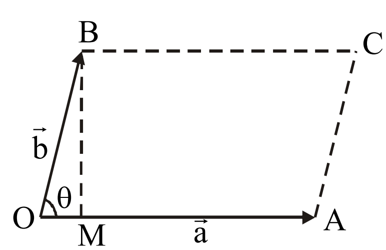
(vi) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 0$



(vii) $\qquad \hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}$

and $\qquad \hat{j} \times \hat{i} = -\hat{k},\ \hat{k} \times \hat{j} = -\hat{i},\ \hat{i} \times \hat{k} = -\hat{j}$

(viii) Area of parallelogram OACB



$= OA \times BM$

$= ab \sin\theta$

$= |\vec{a} \times \vec{b}|$

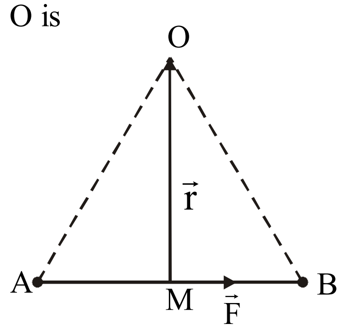
(ix) (a) Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

(b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are position vectors of A, B and C respectively, then area of

$$\Delta ABC = \frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|$$

(x) **Moment of a Force about a Point** Let F be the magnitude of force acting at a point A of the rigid body along the line AB and OM = p is the perpendicular distance of fixed point O from AB, then the moment of force $\vec{F}$ about O is

$$\vec{M} = \vec{F} \times \vec{r}$$



= force x perpendicular distance of he point from the line of action of force

✷ Scalar Triple Product

The scalar triple product of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is denoted by $[\vec{a}\ \vec{b}\ \vec{c}]$.

And defined as $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}.(\vec{b} \times \vec{c})$

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

then $$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Properties of Scalar Triple Product

(i) The value of scalar triple product does not depend upon the position of dot and cross, *ie*,

$$\vec{a}.(\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}).\vec{c}$$

(ii) If $\vec{a}, \vec{b}, \vec{c}$ are cyclically permuted. The value of scalar product ramains same.

The change of cyclic order of vector in scalar triple product changes the sign of the scalar but not the magnitude. *ie*,

$$[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$$

and $$[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{b}\ \vec{a}\ \vec{c}] = -[\vec{c}\ \vec{b}\ \vec{a}] = -[\vec{a}\ \vec{c}\ \vec{b}]$$

(iii) The scalar triple product of three vectors is zero, if any two of them are equal.

(iv) The scalar triple product of three vectors is zero, if two of them parallel or collinear.

(v) If three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanear, then $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

(vi) $[\vec{a}\ \vec{b}\ \vec{c}+\vec{d}] = [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{d}]$

(vii) Scalar triple product geometrically represents the volume of the parallelopiped whose three coterminous edges are represented by $\vec{a}$, $\vec{b}$ and $\vec{c}$, ie, $V = [\vec{a}\ \vec{b}\ \vec{c}]$.

(viii) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar, then $[\vec{a}\ \vec{b}\ \vec{c}] > 0$ for right handed system and $[\vec{a}\ \vec{b}\ \vec{c}] < 0$ for left handed system.

(ix) $[\hat{i}\ \hat{j}\ \hat{k}] = 1$

(x) $[k\ \vec{a}\ \vec{b}\ \vec{c}] = k\,[\vec{a}\ \vec{b}\ \vec{c}]$, Where k is any scalar.

(xi) Volume of tetrahedron with 0 as origin and the position vectors of A, B and C being $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively, is given by $V = \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$.

✱ Vector Triple Product

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors then $\vec{a} \times (\vec{b} \times \vec{c})$represents the vector triple product and is defined as

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}\,.\,\vec{c})\vec{b} - (\vec{a}\,.\,\vec{b})\vec{c}$$

Also, $$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}\,.\,\vec{c})\vec{b} - (\vec{b}\,.\,\vec{c})\vec{a}$$

But $$(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$$

✱ Reciprocal System of Vectors

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vector, if $\vec{a}\,.\,\vec{a}' = \vec{b}\,.\,\vec{b}' = \vec{c}\,.\,\vec{c}' = 1$.

Then are known as reciprocal system of vectors, where $\vec{a}' = \frac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, $\vec{b}' = \frac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, $\vec{c}' = \frac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$.

Properties of Reciprocal System of Vectors

1. Scalar product of any vector of one system with the vector of other system is zero *ie*,

$$\vec{a}\,.\,\vec{b}' = \vec{a}\,.\,\vec{c}' = \vec{b}\,.\,\vec{c}' = \vec{b}\,.\,\vec{a}' = \vec{a}\,.\,\vec{b}' = \vec{a}\,.\,\vec{c}' = 0$$

2. $[\vec{a}\ \vec{b}\ \vec{c}]\,[\vec{a}'\ \vec{b}'\ \vec{c}'] = 1$

3. $\hat{i}' = \hat{i}$, $\hat{j}' = \hat{j}$, $\hat{k}' = \hat{k}$

4. Let $\vec{a}, \vec{b}, \vec{c}$ is a reciprocal system $\vec{a}, \vec{b}, \vec{c}$ and $\vec{r}$ is any vector, then

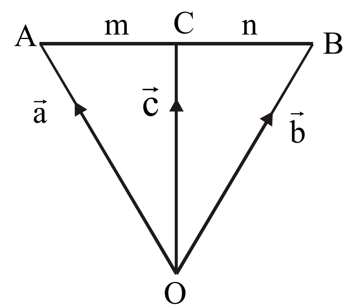
$$\vec{r} = (\vec{r}.\vec{a}')\vec{a} + (\vec{r}.\vec{b}')\vec{b} + (\vec{r}.\vec{c}')\vec{c}$$
$$\vec{r} = (\vec{r}.\vec{a})\vec{a}' + (\vec{r}.\vec{b})\vec{b}' + (\vec{r}.\vec{c})\vec{c}'$$

5. If the system $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar and so are the reciprocal system $\vec{a}', \vec{b}', \vec{c}'$.

✷ Application of Vector in Coordinate Geometry

Section Formula

Let A and B be two points with position vector $\vec{a}$ and $\vec{b}$. Let C be any point dividing AB in the ratio of *m* : *n*. Then



$$\vec{c} = \frac{m\vec{b} + n\vec{a}}{m + n} \qquad \text{(for internal divison)}$$

and

$$\vec{c} = \frac{m\vec{b} - n\vec{a}}{m - n} \qquad \text{(for external divison)}$$

Equation of Straight Line in Vector Form

1. Equation of straight line passing through the point $\vec{a}$ and parallel to vector $\vec{b}$ is $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ or $\vec{r} = \vec{a} + t\vec{b}$.
2. Equation of straight line passing through the two points $\vec{a}$ and $\vec{b}$ is $(\vec{r} - \vec{a}) \times (\vec{b} - \vec{a}) = 0$ or $\vec{r} = (1 - t)\vec{a} + t\vec{b}$.
3. Equation of straight line passing through the point $\vec{a}$ perpendicular to two non-parallel vectors $\vec{c}$ and $\vec{d}$, is $(\vec{r} - \vec{a}) \times (\vec{c} \times \vec{d}) = 0$

Equation of a Plane in Vector Form

1. Equation of a plane passing through the point a and parallel to non-parallel vectors b and c is $[\vec{r}\,\vec{b}\,\vec{c}] = [\vec{a}\,\vec{b}\,\vec{c}]$ or $\vec{r} = \vec{a} + t\vec{b} + s\vec{c}$.
2. Equation of plane passing through the three point $\vec{a}$, $\vec{b}$ and $\vec{c}$ is

$$\vec{r}.(\vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}) = [\vec{a}\,\vec{b}\,\vec{c}]$$

and

$$\vec{r} = \vec{a} + t(\vec{b} - \vec{a}) + s(\vec{c} - \vec{a}).$$

3. Equation of plane which passes through the point a and perpendicular to $\vec{n}$, is $\vec{r}.\vec{n} = \vec{a}.\vec{n}$.

Equation of a Plane Containing the Line of Intersection of Two Planes

Let two planes be $\vec{r}.\vec{n}_1 = q_1$ and $\vec{r}.\vec{n}_2 = q_2$, then equation $(\vec{r}.\vec{n}_1 - q_1) + (\vec{r}.\vec{n}_2 - q_2) = 0$, Where λ be any scalar quantity, is the equation of plane passing through the intersection line of planes.

Equation of a Line of Intersection of Two Planes

Let two planes be $\vec{r}.\vec{n}_1 = q_1$ and $\vec{r}.\vec{n}_2 = q_2$, be two equation of planes, then the equation of a line of intersection of two planes, is $\vec{r} = \vec{a} + t(\vec{n}_1 \times \vec{n}_2)$, where t be any scalar.

Note :

- If $\vec{a}$ and $\vec{b}$ are two vectors, then the subtraction of $\vec{b}$ from a is defined as the vector sum of $\vec{a}$ and $-\vec{b}$, ie, reverse the direction of $\vec{b}$ and add it to $\vec{a}$.
- If two vectors a = (a_1, a_2, a_3) and b = (b_1, b_2, b_3) are equal, their resolved parts will also equal ie, $a_1=b_1$, $a_2=b_2$ and $a_3=b_3$.
- The resolved parts of a resultant vector of addition of two vectors are equal to the sum of resolved parts of those vectors.
- Three vectors $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar, if

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

- The vector component of $\vec{a}$ along $\vec{b} = \left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ and perpendicular to $\vec{b} = \vec{a} - \left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.
- Any vector a can be written as, $\vec{a} = (\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k}$